2024 Lecture in Memory of Nora Szech

Nikolaus Schweizer
Tilburg University

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Remembering Nora

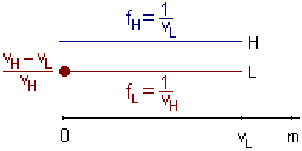
- I first met Nora in late 2002 in a seminar on Gottlob Frege's philosophy of language and knew her well in the twenty years that followed.
- We played in the same folk rock band, had endless discussions about pretty much everything and, at some point, we even started coauthoring research papers in theoretical economics.
- This presentation is a journey through some of my favorites from Nora's research, including some of our joint work.

Tie-break and bid-caps in all-pay auctions

GEB 2015

All-pay auctions with and without bid-caps

- Consider a two bidder all-pay auction with complete information and valuations $v_H > v_L > 0$.
- Classical equilibrium involves both bidders mixing over $[0, v_L]$ and the weak bidder placing an atom in 0.



- Che and Gale (AER 1998): By imposing a maximum bid (a "cap") of $m = \frac{v_L}{2}$ a designer can increase the expected sum of bids, creating a situation where both bidders bid $\frac{v_L}{2}$ and win with probability $\frac{1}{2}$.

Tie-breaks and bid-caps

- Nora: By breaking ties symmetrically, the designer leaves some willingness to pay on the table... There is room for raising the maximum bid m if the designer breaks ties in favor of the weaker bidder.

- Denote by α the tie-breaking probability of the stronger bidder. Then the combination of m and α that maximizes the expected sum of bids solves

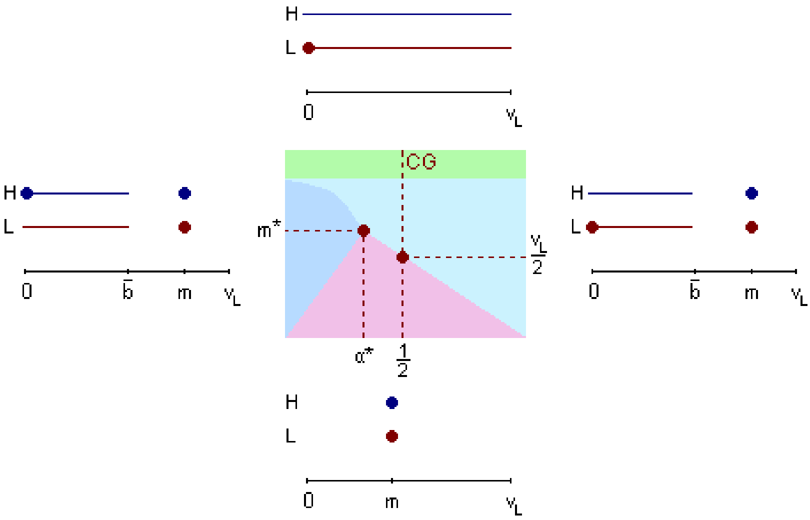
$$\alpha v_H - m = 0 \quad \text{and} \quad (1-\alpha)v_L - m = 0$$

  generating a pure strategy equilibrium in which both bidders make zero profit.

- Solution:

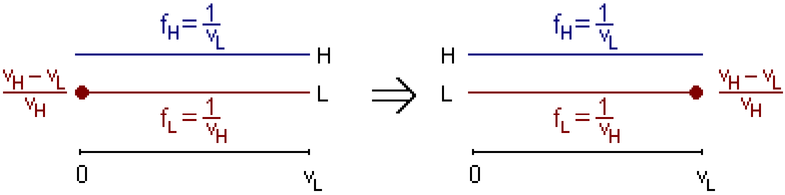
$$\alpha * = \frac{v_L}{v_H + v_L} \quad \text{and} \quad m^* = \frac{v_H v_L}{v_H + v_L} > \frac{v_L}{2}$$

The full equilibrium landscape



The upper left corner

- The combination that maximizes the expected maximum bid is $\alpha = 0$, $m = v_L$.
- Compared to the unrestricted auction, this takes away the strong bidder's chance to slightly overbid the weak bidder at the top.



- Weak bidder becomes indifferent whether to place atom at the top or bottom.
- For $\alpha = 0$ and $m = v_L - \varepsilon$, equilibrium with atom at the top becomes unique.

Optimal revelation of life-changing information

Schweizer & Szech, MS 2018

The idea

- Suppose there is a 50/50 chance that you suffer from a severe disease that may or may not break out in 15 years.
- There is the possibility to genetically test whether you are affected. This is purely informative from a medical point of view. It has no impact for a possible treatment and there is no danger of infecting others.
- Consider a testing mechanism that informs you about the healthy outcomes in 50% of the scenarios in which you are healthy. Otherwise it sends you a noisy signal. Many people would prefer such a noisy test over a perfectly informative one.
- In the bad outcome, the odds become 66/33 so a bit worse than 50/50 – but there is never a test result that provides a confirmation of carrying the disease... and with some probability you are free.

Some comments

- To turn this intuition into a paper, we needed a model... we combined anticipatory utility, i.e.,

$$E\left[U\left(\text{Prob}\{\text{being healthy} \,|\, \text{test result}\}\right)\right]$$

  with a cost term which captures that more precise information enables people to plan their life better.

- Roughly speaking, $U'' < 0$ gives information avoidance, the costs introduce a tradeoff and $U''' > 0$ gives an asymmetry between good and bad news.
- Ex post, this was one of the first few Bayesian persuasion papers... even though we didn't know this when we worked on it in 2011/12. For some time, it was one of the few that started out from a somewhat applied motivation.
- Is this behavioral economics?

Optimal advertising of auctions

JET 2011

The setting

- Consider an auctioneer who faces a convex and increasing cost $c(n)$ for attracting n bidders to an SIPV second price auction.
- A welfare maximizer solves

$$\max_n E[X_{1:n}] - c(n)$$

while a revenue maximizer solves

$$\max_n E[X_{2:n}] - c(n)$$

- Who will end up choosing a higher n^*?
Idea: If $\Delta_n = E[X_{1:n} - X_{2:n}]$ is increasing in n then

$$E[X_{1:n+1} - X_{1:n}] \geq E[X_{2:n+1} - X_{2:n}]$$

for all n, so a welfare maximizer will choose a larger auction. And vice versa.

Spacings of order statistics I

- Lemma: If $E[|Y|] < \infty$ then $E[Y_{1:n}]$ is increasing and concave while $E[Y_{n:n}]$ is decreasing and convex.
- Denote by F and f the pdf and cdf of X_i. Writing

$$\Delta_n = E[X_{1:n} - X_{2:n}] = E[h(X_{1:n})] \text{ with } h(x) = \frac{1 - F(x)}{f(x)}$$

  shows that Δ_n is decreasing if h is decreasing because then $E[h(X_{1:n})] = E[Y_{n:n}]$ where $Y_i = h(X_i)$.
- Similarly, Δ_n is increasing if h is increasing.
- $1/h$ is the so-called failure rate or hazard rate... Condition is also known as MHR or IFR/DFR.

Spacings of order statistics II

- The welfare maximizer's objective is always concave because $E[X_{1:n}]$ is concave and $c(n)$ is convex. But when is $E[X_{2:n}]$ concave?
- One can write

$$E[X_{2:n+1} - X_{2:n}] = E[h(X_{1:n})] \text{ with } h(x) = \frac{(1 - F(x))^2}{f(x)}$$

- Thus, whether expected second order statistics are concave or convex depends on whether the function $f/(1-F)^2$ is increasing or decreasing. If $f/(1-F)^2$ is increasing the problem is concave.
- Can $E[X_{2:n}]$ really be strictly convex if $E[X_{1:n}]$ is concave?
- Yes, but only if $E[X] = E[X_{1:n}] = \infty$.

Performance bounds for optimal sales mechanisms beyond the monotone hazard rate condition

aka

The quantitative view of Myerson regularity

Schweizer & Szech, JMathE 2018

Starting point

- How is monotonicity of $f(x)/(1-F(x))^2$ related to Myerson regularity, i.e., to monotonicity of

$$x - \frac{1-F(x)}{f(x)} \, ?$$

- Turns out the two conditions are equivalent ...
- Let's call a distribution λ-regular if

$$r_\lambda(x) = \frac{f(x)}{(1-F(x))^{1+\lambda}}$$

  is increasing for some $\lambda < 1$. What does this mean and imply?
- Interpolates between Myerson regularity, IFR and monotonicity of the density. Has been studied in various literatures under different names e.g. α-strong regularity, ρ-concavity etc...

Some implications

- Monotonicity of $r_\lambda(x)$ is equivalent to monotonicity of $\lambda x - \frac{1-F(x)}{f(x)}$.
- Any continuous distribution can be written as

$$1 - F(x) = \Gamma_\lambda \left(\int_x^\infty r_\lambda(y) dy \right)$$

  where $\Gamma_\lambda(z) = (1 + \lambda z)^{-1/\lambda}$ is decreasing.
- Let F be λ-regular and denote by p^* the solution to $p^* - \frac{1-F(p^*)}{f(p^*)} = 0$. Then $P(X > p^*) \geq (1-\lambda)^{1/\lambda}$.
- If F is λ-regular with support unbounded from above then

$$\lim_{n\to\infty} \frac{E[X_{2:n}]}{E[X_{1:n}]} \geq 1 - \lambda$$

- Lower bounds vanish for $\lambda = 1$...

Revenues and Welfare in Auctions with Information Release

Schweizer & Szech, JET 2017

Information Release in Auctions

- How does information release affect expected differences of order statistics? And thus the difference between welfare and revenue in an SIPV auction model

$$E[X_{1:n} - X_{2:n}] = \int_0^1 x^{n-1}(1-x)dF^{-1}(x)$$

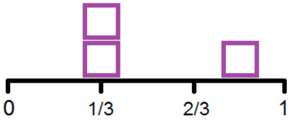
- Denote by F and G the cdf of valuations with and without release of additional information.
- Classical approach: Assume "$dF^{-1}(x) \geq dG^{-1}(x)$" for all x.
- All pairs of quantiles lie further apart under F than under G. This is the dispersive order, $F \succeq_{disp} G$.

Theorem (Ganuza and Penalva, E'trica 2010)

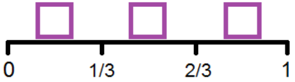
If $F \succeq_{disp} G$ then a welfare maximizer has a stronger incentive to release information than a revenue maximizer.

Example

- Bidders' unknown true valuations are uniformly distributed on $[0, 1]$.
- Initially, each bidder only knows whether her valuation lies above or below $2/3$.
- Thus, G places a mass of $2/3$ on $1/3$ and $1/3$ on $5/6$.

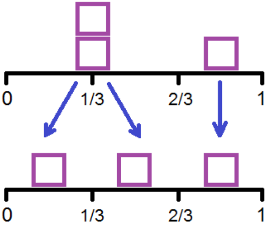


- Under information release, each bidder additionally learns whether her valuation is above or below $1/3$.
- Thus, F is the uniform distribution on $1/6$, $1/2$ and $5/6$.



Information Release and Dispersion Criteria

- In examples like this – thus, e.g., for all kinds of information partitions – the dispersive order is violated.
- Information release pushes some quantiles more closely together.



- In this example, a welfare maximizer has a weaker incentive to release information than a revenue maximizer for $n > 3$, and vice versa.

What did I not talk about

- Mice
- Nora's other experimental work, e.g., work on
  - Updating, self-confidence and discrimination
  - Ethical consumption
  - (In)elasticity of moral ignorance
- Most of her earlier theory work, e.g. most of her dissertation, but also some of the later... like her work on matching with Benny and Deniz, or our joint paper on shared responsibility and moral transgression in public goods games...

Thank you!

And before I forget: We are editing a special issue of the European Economic Review in Nora's memory. Deadline is December 16 2024.